PROBLEMS OF SUPERSONIC FLOW OVER BODIES OF PRISMATIC CONFIGURATION

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In problems of flow of a supersonic stream ($M > 1$) over bodies with a spatial configuration which slightly perturbs the incident flow, the gasdynamic equations can be reduced to a wave equation for the perturbation velocity potential in the form [1, 2]

$$F(\Phi) = \Phi_{xx} - \Phi_{yy} - \Phi_{zz} = 0,$$

where the x axis is coplanar with the velocity vector of the incident flow. The perturbed flow region is limited by the body surface and the leading characteristic of the surface, which is the envelope of characteristic cones whose peaks lie on the supersonic portion of the body's leading edge.

In considering problems of flow over an isolated wing of finite size in the linear formulation, the conditions on the wing surface are referred to a base surface, which is a plane which deviates only slightly from the wing surface [3]. In considering flow over bodies with the configuration of an airplane (wing–fuselage–air scoops) for the base surface we choose cylindrical surfaces, in particular, prismatic configurations with edges parallel to the incident flow, such that the faces of these prismatic configurations deviate only slightly from the slightly curved aircraft surfaces (Fig. 1). The base surface to which the boundary conditions on the body are referred is a surface of the time-oriented type. The value of the potential at the point $M(x, y, z)$ in the perturbed flow region depends on the initial data for only that part of the boundary manifold (the leading characteristic surface σ and the base surface S) which is located within the characteristic cone Γ: $(x-\xi)-[(y-\eta)^2+(z-\zeta)^2]^{1/2}=0$.

If the region of dependence D of the point M has the property of visibility (that is, if any point of the region of dependence can be joined to the point M by a straight line lying wholly within the region of dependence), then the line of intersection of the characteristic cone Γ with the boundary manifold σ, S defines the region of dependence of the point M on this manifold. The condition of visibility in the region of dependence D is satisfied in the case where for points of the perturbed flow region on the body surface there are no points located within the shadow zone, i.e., when the straight line passing through the point M within the characteristic cone intersects the body surface at not more than one point. If these exists a plane T passing through point M and tangent to the body surface, separating in space a visibility zone and a shadow zone for point M, then the region of dependence of point M ceases to have the property of visibility. On the body surface the boundary of the visibility zone and the shadow zone for point M is the line of tangency t of the plane T on the body surface (line AA on the cylindrical base surface of Fig. 2). The region of dependence of point M on the body in the visibility

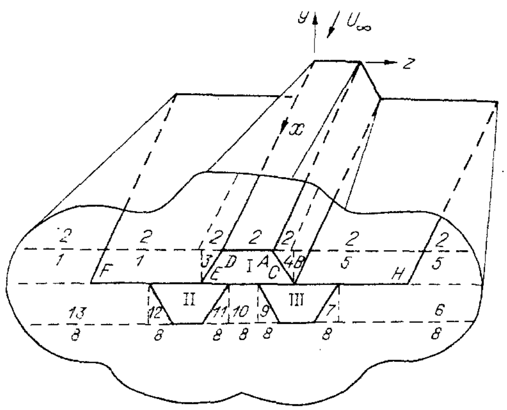

Fig. 1

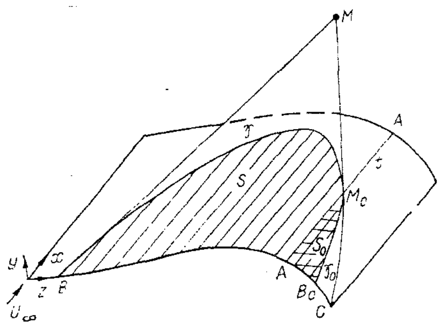

Fig. 2

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zone is bounded by the line γ on which the characteristic cone Γ intersects the body surface, while in the shadow zone the region of dependence is limited by the spiral line γ_0, which has a characteristic direction everywhere on the body surface. The spiral line $\gamma_0 (M_0B_0)$ passing through the point of intersection M_0 of the lines γ and t. The region of dependence of the point M in the visibility zone lies within the surface of the characteristic cone Γ, while in the shadow zone the region lies within the surface Γ_0, which is formed by the envelope of cones with vertices on the spiral line γ_0. The surface Γ_0 is tangent to the surface Γ along a line lying in the plane of the shadow boundary T. The potential value at the point M depends on the initial data of only that portion of the boundary manifold (body surface and leading characteristic surface) which is located within the characteristic surface Γ in the visibility zone and within the surface Γ_0 in the shadow zone. The portion of the boundary manifold in the shadow zone located between the surfaces Γ and Γ_0 has no effect on the potential at point M. On the body, this is the region M_0B_0C, limited by the leading edge of the body, the spiral line M_0B_0, and the line of intersection of the characteristic cone Γ with the body surface in the shadow zone M_0C. For a prismatic configuration with dihedral angle $\pi \leq \gamma \leq 2\pi$, the surface Γ_0 is a characteristic cone with apex at the point M_0 [4]. In [5, 6], which considered flow over bodies with shadow zones, the solution at point M was presented as dependent on boundary values on a portion of the body surface in the shadow zone lying outside the actual region of dependence of the point M. This discrepancy was noted in [7], but no method of solving the diffraction problem was suggested.

For the region of dependence D having the property of visibility, using Green's formula

$$\iiint_D [wF(\Phi) - \Phi F(w)]\, d\tau = I_{\Gamma+\sigma+S}^{w,\Phi'_N,\Phi,w'_N}, \tag{1}$$

$$I_{\Gamma+\sigma+S}^{w,\Phi'_N,\Phi,w'_N} = \iint_{\Gamma+\sigma+S} \left(w\frac{\partial \Phi}{\partial N} - \Phi \frac{\partial w}{\partial N} \right) dS,$$

$$w = \ln \frac{(x-\xi) - \sqrt{(x-\xi)^2 - (y-\eta)^2 - (z-\zeta)^2}}{\sqrt{(y-\eta)^2 + (z-\zeta)^2}},$$

an integral representation of the solution in Volterra form can be obtained [8, 1, 2, 9]

$$\Phi(M) = \frac{1}{2\pi} \frac{\partial}{\partial x} I_{\Gamma+\sigma+S}^{w,\Phi'_N,\Phi,w'_N}, \tag{2}$$

where w is the fundamental Volterra solution; $\partial/\partial N$, conormal derivative $\Phi'_N = \partial\Phi/\partial N$, $w'_N = \partial w/\partial N$); Γ, portion of the characteristic cone from the apex at point M to the intersection with the boundary manifold; S, σ, portions of the surfaces of the body and the leading characteristic cone cut by the characteristic cone Γ.

For the region of dependence D which does not have the property of visibility, we may formally write an integral representation of the potential value at point M in Volterra form, applying Green's formula (1) to this region:

$$\Phi(M) = \frac{1}{2\pi} \frac{\partial}{\partial x} I_{\Gamma+\Gamma_0+\sigma+\sigma_0+S+S_0}^{w,\Phi'_N,\Phi,w'_N}. \tag{3}$$

Here Γ, Γ_0 are the portion of the surface of the characteristic cone with apex at point M in the visibility zone and the portion of the envelope of cones with apexes on the spiral line γ_0; σ, σ_0 are the portion of the leading characteristic surface cut by the cone Γ and the surface Γ_0; S, S_0 are the regions of dependence on the body in the visibility zone and the shadow zone.

On the right side of Eqs. (2), (3), in view of the properties of the Volterra function the integral over the surface Γ vanishes. On the leading characteristic surface σ, σ_0 the value of the perturbation potential Φ can be taken equal to zero ($\Phi|_\sigma = 0$) without limiting generality, since on the surface σ, σ_0 the direction of the derivative $\partial/\partial N$ coincides with the direction of the tangent to the line z = const of this surface, which means that on the surface σ, σ_0, along with the value of Φ the value of $\partial\Phi/\partial N$ is also defined and equal to zero, so that the integral over the surface σ, σ_0 in Eqs. (2), (3) vanishes. While in Eq. (2) there remains only the integral over the body surface S, in Eq. (3) together with the integral over the body surface $S + S_0$, there also remains the integral over the surface Γ_0, which in view of the properties of the Volterra function does not vanish. This means that in Eq. (3) the potential at the point M is determined not only by the values of Φ, Φ'_N on the body, but also by the values of Φ, Φ'_N unspecified on the surface Γ_0 and the integral representation of Eq. (3) is not a solution of the boundary problem analogous to that of Eq. (2) for the zone of dependence possessing the property of visibility.

The problem of flow over bodies with regions of dependence not having the visibility property is within the class of diffraction problems: The character of the action on point M of points of influence lying within the shadow zone differs from the character of action of points of influence lying within the visibility zone. While for the visibility zone we may use a solution in the Volterra form, reflecting the character of the direct action of points of influence on the point in question, for solution of the diffraction problem it is necessary to find some solution of the wave equation which will reflect the diffraction character of the action of points of influence on that point. For the case of an arbitrary dihedral angle $\pi \le \gamma \le 2\pi$ [4] found a fundamental solution v_0 which considers the diffraction character of the phenomenon.

In Eq. (2) for the visibility region, in view of the assumption that $\Phi|_\sigma = 0$ without limiting generality, and the properties of the Volterra function, the potential at the point is expressed solely in terms of integrals over the body surface S:

$$\Phi(M) = \frac{1}{2\pi}\frac{\partial}{\partial x} I_S^{w, \Phi_n', \Phi, w_N'}. \tag{4}$$

Then, since conditions on the body surface are referred to base surfaces of a time-oriented type, where the conormal derivative of the potential Φ'_N coincides with the normal derivative Φ'_n to the accuracy of small second-order terms, Eq. (4) provides a representation of the perturbation potential in terms of the values of $\Phi|_S$ and $\Phi'_n|_S$.

On the time-oriented surfaces there exists a relationship between the value of the potential $\Phi|_S$ and its normal derivative $\Phi'_n|_S$, defined in the general case of a cylindrical base surface by an integrodifferential expression, which can be obtained from Eq. (4) in the limit as the point M approaches the surface S (see, for example, [2]).

In the case of a region, the base surfaces of which are an arbitrary dihedral angle $0 < \gamma < \pi$, where $\gamma \neq \pi/n$ (n=1, 2, ...), the values of $\Phi|_S$ unspecified in the direct problem (the problem of determining the velocity potential from the body geometry) on the face surfaces are defined in terms of the values of $\Phi'_n|_S$ from a system of two integrodifferential equations by the method of successive approximation [10].

For some prismatic regions, by using the method of compensating singularities it is possible to find a superposition of fundamental solutions (Green's functions) in which in Eq. (4) for the velocity potential terms containing values of $\Phi|_S$ on the faces are excluded in the direct problem, and the value of the potential is expressed solely in terms of the values of the normal derivative of the potential $\Phi'_n|_S$ on the faces, or in Eq. (4) for the velocity potential terms containing values of $\Phi'_n|_S$ on the faces are excluded in the converse problem.

Thus, for base surfaces formed by a dihedral angle $\gamma = \pi/n$ (n=1, 2, ...), the integrals in Eq. (4) from terms containing the value of the potential $\Phi|_S$ can be eliminated by using the method of reflection from the faces of the dihedral angles for compensating singularities of the Volterra type [1, 2, 11], finally obtaining for the potential Φ in the perturbed region an expression solely in terms of values of $\Phi'_n|_S$ specified on the dihedral faces:

$$\Phi_1(M) = \frac{1}{2\pi}\frac{\partial}{\partial x}\left(I_{S_1+S_2}^{w, \Phi_n'} + \sum_{i=1}^{2n-1} I_{S_{i1}+S_{i2}}^{w_i, \Phi_n'}\right), \tag{5}$$
$$I_{S_1+S_2}^{w, \Phi_n'} = \iint\limits_{S_1+S_2} w\Phi_n' dS, \quad I_{S_{i1}+S_{i2}}^{w_i, \Phi_n'} = \iint\limits_{S_{i1}+S_{i2}} w_i \Phi_n' dS.$$

Here w, w_i are Volterra functions of the point M and the compensating point M_i; S_1, S_2, S_{i1}, S_{i2} are the regions of dependence of points M, M_i on faces 1, 2 of the dihedral angle. Or, for the same base surfaces formed by a dihedral angle $\gamma = \pi/n$ (n=1, 2, ...), one can also use the reflection method (again employing the parity properties of the functions w, w_i, and the nonparity of the functions w'_N, w'_{iN} relative to the corresponding face of the dihedral angle) to eliminate from Eq. (4) terms containing values of $\Phi'_n|_S$, and obtaining finally an expression for the potential Φ in the perturbed region solely in terms of values of $\Phi|_S$ on the faces of the dihedral angles:

$$\Phi_2(M) = -\frac{1}{2\pi}\frac{\partial}{\partial x}\left(I_{S_1+S_2}^{w_N', \Phi} - \sum_{i=1}^{2n-1} I_{S_{i1}+S_{i2}}^{w_{iN}', \Phi}\right), \tag{6}$$
$$I_{S_1+S_2}^{w_N', \Phi} = \iint\limits_{S_1+S_2} w_N' \Phi dS, \quad I_{S_{i1}+S_{i2}}^{w_N', \Phi} = \iint\limits_{S_{i1}+S_{i2}} w_N' \Phi dS.$$

Here w'_N, w'_{iN} are conormal derivatives of the Volterra function of the point M and the compensating point M_i.

If we differentiate the integral operators I with respect to x (in Eq. (6) we must first integrate by parts over ξ), it can be shown, as in [12], for the case of an isolated wing (n=1), that Eq. (5) gives a solution to the direct problem of determining the velocity potential from the normal derivative Φ'_n on the body surface, while Eq. (6) provides a solution of the converse problem of determining the velocity potential from a pressure distribution Φ'_ξ specified on the body surface.

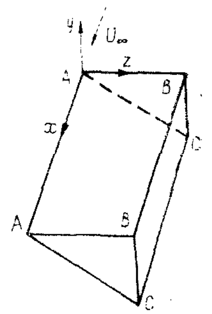

Fig. 3

For a dihedral angle $\gamma = \pi/n$ (n = 1, 2, ...) the number of compensating points in the superposition for the face interaction region is independent of the depth of the location of the point under consideration within the flow, and for points not lying on a face is equal to (2n−1).

For a region in the form of a band, limited by a combination of two parallel faces ($\pi/2$, $\pi/2$), for a region in the form of a semiband limited by three faces, for the internal region of a three-faced prism ($\pi/2$, $\pi/4$, $\pi/4$), ($\pi/3$, $\pi/3$, $\pi/3$), ($\pi/2$, $\pi/3$, $\pi/6$) and the internal region of a four-faced prism ($\pi/2$, $\pi/2$, $\pi/2$, $\pi/2$) a construction using the Green function compensating singularities is also possible [2, 13]. The number of compensating points for these superpositions increases down the flow as a function of the number of reflections from the faces of the characteristic surfaces of the original point M and the compensating points M_i. The principle of Green's function construction depending on the number of reflections for the direct problem in a band was presented in [2]. Location of compensating points and Green's function construction are performed similarly in the direct and converse problems for the other prismatic regions with larger number of reflecting faces referred to above. We will illustrate this with the example of the direct problem within a three-faced prism ($\pi/6$, $\pi/3$, $\pi/2$), the edges of which are parallel to the incident flow velocity (Fig. 3). For such a prism Fig. 4 shows the locations of compensating points in the plane $\xi = x$ for the point M(x, 0, z), located on the face AB near the dihedral angle $\gamma_A = \pi/6$. In the case where the face BC still has no effect on the point M, the compensating points for the dihedral angle γ_A are located on a circle of radius r = z with center at the point A(x, 0, 0). The point M itself is denoted by the digit 0, while the compensating points of the angle $\gamma_A = \pi/6$ are denoted by digits 1-5. When the face BC begins to affect point M, to compensate reflections from the face BC we locate as a mirror image of the circle with center at point A another circle with center at A_1 (A−BC) with corresponding compensating points 01-51. When the characteristic cones of points 01-51 begin to intersect the face AC, then for compensation in the Volterra expression for the direct problem of terms with $\Phi w'_N$ from these points on the face AC we generate a mirror reflection of the circle with center A_1 relative to the line AC in the form of a circle with center at the point A_2(A_1−AC), A_3(A_2−AB), A_4(A_3−AC), A_5(A_4−AB), A_6(A_4−BC), A_7(A_6−AB, A_5−BC), A_8(A_5−AC). Figure 4 shows a construction of compensating points up to production of a

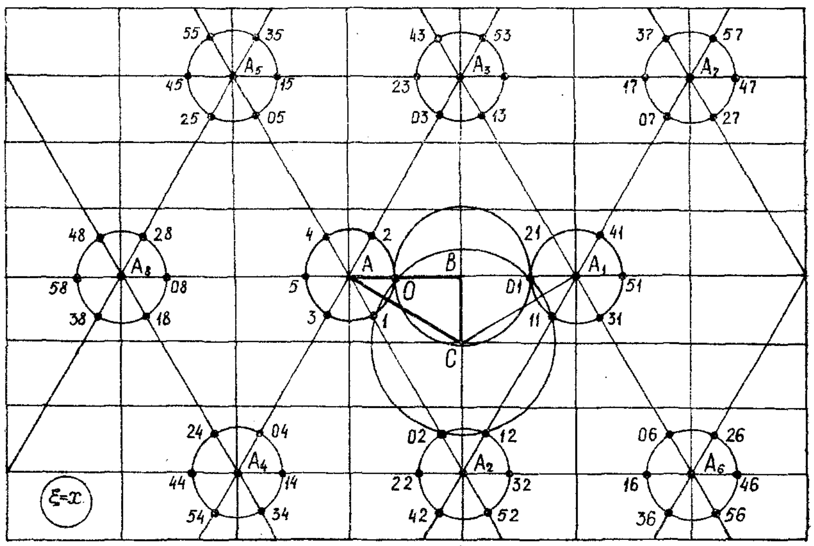

Fig. 4

circle with center at the point $A_{8(6+2)}$, when the positions of compensating points 08–58, corresponding to the original circle with center at A, are repeated. After this the compensation point construction begins with the same sequence as in the $A-A_8$ cycle; $A_9(A_8-BC)$, etc. A similar periodicity occurs with other prisms, for example, a three-face prism ($\pi/4$, $\pi/4$, $\pi/2$) causes repetition of the situation corresponding to initial location of the point M near the angle $\pi/4$ beginning with the reflection $A_6(4+2)$.

For regions having the property of visibility, the boundary of which is a dihedral angle $\gamma=(m/n)\pi$, the method of compensating singularities cannot be applied directly, since in this case one of the compensating points falls in the region of the dihedral angle $\gamma=(m/n)\pi$. In this case the dihedral angle $\gamma=(m/n)\pi$ is divided into $l \leq m$ dihedral angles $\gamma_k=\pi/k$ ($k \leq n$), to each of which the compensating singularity method is applicable. On the planes dividing the dihedral angle into subregions γ_k the solutions are merged together, employing the continuity of the potential and its normal derivative. Solution of the direct problem in this case reduces to solution of a system of generalized Abel equations [14].

In solving problems of flow over complex regions not having the property of visibility, when both direct action and diffraction phenomena occur, the complex region can be divided into subregions, each of which has the property of visibility, with subsequent merger of the solutions on the division boundaries using the continuity of the potential and its normal derivative. In the case of prismatic base configurations one usually constructs planes which are continuations of the faces of the base prismatic configuration, and then solutions are merged on the portions of these planes lying in the perturbed flow region ("slots").

In problems dealing with end effects of isolated wings [3], cross-shaped wings with dihedral angles between wings of $\gamma=\pi/n$ [11], parallel wings [2], and dihedral angles $\gamma=(m/n)\pi$ [14] the merger is performed on slits which are a continuation of the wing base planes.

In problems of flow over a prismatic configuration representing the base surface of an aircraft configuration of the wing–fuselage–air scoop type, when the faces of the base configurations in general lie in various noncoplanar planes, division into subregions having the visibility property cannot be performed by a single method. The division must be performed so that the number of merger planes is minimal and so that the subregions are simple in the sense of finding a superposition of fundamental Volterra solutions.

Figure 1 shows an example of the prismatic base configuration for a wing with superstructures I–III of the fuselage–air scoop type, with division of the external flow region into 13 simple subregions having the visibility property, the faces of which form dihedral angles π/n and for each of which a Green function can be constructed. We will consider the direct problem. Green's function defines the potential at some point of the subregion in terms of the values of the normal derivative of the potential on the faces which are boundaries of the subregion. On the faces which are boundaries of the base configuration the values of Φ_n' are known, while on the slot-boundaries the values of $\Phi_n'=\theta$ are defined by the merger conditions. As an illustration we will construct the merger condition for a point lying on the face AB, dividing regions 2 and 4 (Fig. 1). Subregion 2 is bounded by the leading characteristic surface σ and the plane $y=0$ passing through face DA of superstructure 1, while subregion 4 is the inner region of a three-faced prism ABC (let $\gamma_A=\pi/6$, $\gamma_B=\pi/2$, $\gamma_C=\pi/3$). The potential at the point M, lying on the boundary-slot AB, can be written in subregion 2 as

$$\Phi(M_+)=\frac{1}{\pi}\iint\limits_{S_M}\Phi_n' r_0^{-1/2}\,dS.$$

Here r_0 is the hyperbolic distance of the point M(x, 0, z) to points of the region S_M, which is the region on the plane $\eta=0$ bounded by the line of intersection of the characteristic cone of the point M with the plane $\eta=0$ and the line $\xi=0$ (the triangle MVW, Fig. 5). In subregion 4, using Green's function construction for that region, the potential at the same point M can be represented as

$$\Phi(M_-)=\frac{1}{\pi}\sum\iint\limits_{S_{ijk}}\Phi_n' r_{ij}^{-1/2}\,dS. \tag{7}$$

Here r_{ij} is the hyperbolic distance from the compensating point M_{ij} to points of the region S_{ijk}, which is a region on one of the faces of prism ABC, bounded by the line of intersection of the characteristic cone of compensation point M_{ij} with the prism face k, the edges of this face, and the line $\xi=0$ on this face. Figure 5 shows the situation in which the face BC now has an effect on the point M, lying on the face AB in the sphere of action of the angle $\gamma_A=\pi/6$, and in construction of the Green function, aside from the point M itself (point 0) and compensating points M_1-M_5 (points 1-5) there participate points M_{01}, M_{11}, M_{21} (points 01, 11, 21). In the lower left corner of Fig. 5 the lines of intersection of characteristic cones with apexes at the points M_0-M_5, $M_{01}-M_{21}$ with the plane $\eta=0$ (region S_{ij} on face AB) are shown; the upper right corner shows lines of intersection of

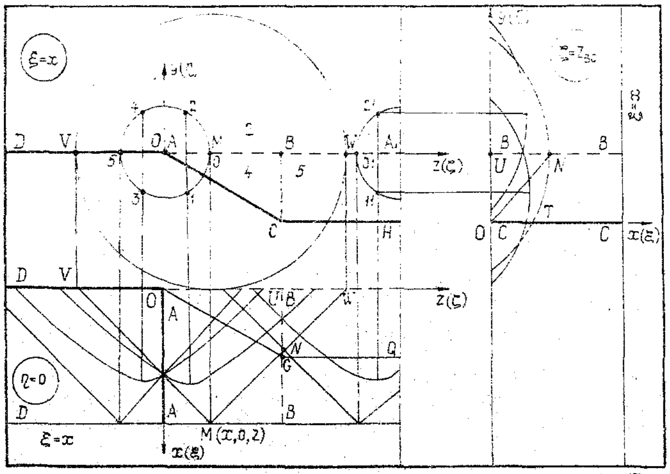

Fig. 5

characteristic cones with apexes at the points M, M_1, M_2, M_{01}, M_{11}, M_{21} with the plane $\zeta = z_{BC}$ (region S_{ij} on face BC). On the faces AD, AC, which are faces of the base configuration, the values of Φ_n' are known, while on the slot faces AB, AC the value of $\Phi_n' = \theta$ is subject to definition from the condition $\Phi(M_+) = \Phi(M_-)$, which for $\theta|_{AB}$ can be represented as a generalized Abel equation [14]:

$$A(\theta|_{AB}) + \sum B_{ij}(\theta|_{AB}) = F_1(\Phi_n'|_{AD}, \Phi_n'|_{AC}) + F_2(\theta|_{BC}). \tag{8}$$

Here the term $A(\theta)$ is an Abel operator and corresponds to the term in Eq. (7) for the region S_M on the face AB with integrand $r_0^{-1/2}$ for the point M; the term $B_{ij}(\theta)$ corresponds to the term in Eq. (7) for the region S_{ij} on the face AB with integrand $r_{ij}^{-1/2}$ for the point M_{ij}, with the region S_{ij} included within the region S_M. On the right side of Eq. (8), aside from the term F_1, which is a function of the known quantities $\Phi_n'|_{AD}$, $\Phi_n'|_{AC}$, we have the term F_2, which is a function of the previously unknown quantity $\theta|_{BC}$ on the face BC. To determine $\theta|_{BC}$ we also use the condition $\Phi(M_+) = \Phi(M_-)$, written for a point M lying on the face BC, which is a boundary of the subregions 4 and 5. In the general case, the merger conditions for all subregions form a closed system of generalized Abel equations of the form of Eq. (8) for determination of the θ values on the slot-faces of the configuration. Definition of θ values on these faces is performed sequentially down the flow, commencing from regions of simple flows to regions where multiple reflections from the faces and diffraction on the edges begin to have an effect. For example, there are regions on each of the faces AB and BC where the flow is still unperturbed, $\theta|_{AB} = \theta|_{BC} = 0$ (the line OGQ is the track of the leading characteristic surface in the plane $\eta = 0$); further down the flow on the face AB there is a region of diffraction from the dihedral angle γ_A, in which reflection from the face CH still has no effect and the value of $\theta|_{AB}$ can be determined as the effect of elementary diffraction on the angle, etc. As we move down the flow, new regions on the faces with undetermined values of θ_{ij} are included, and these values are found from known values of Φ_n' on the faces of the base configuration and values of θ_{ij} determined previously in higher-lying regions. The principle of sequential determination of θ_{ij} values down the flow was described in [3, 2, 14].

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GAS CAVITY DYNAMICS IN A CONTACT UNDERWATER ELECTRICAL EXPLOSION

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A contact electrical explosion is often used for the electrohydroimpulse destruction of materials [1], when one of the electrodes is the workpiece. This means that the vapor–gas cavity which is formed develops directly on the object being processed (the solid wall), so the latter may affect both the pressure field formed in the liquid and the manner in which the electrical explosion develops.

In the majority of theoretical investigations on the dynamics of cavities in contact with a solid wall [2–5], the problem of the collapse of a spherical cavity [2, 3] or an ellipsoidal cavity [4, 5] is considered as the model problem. However, as experiment shows [6–10], because of the initial increase in the cavity under asymmetric boundary conditions it is subject to deformations. The changes in shape which therefore occur may be extremely important [6, 8], and, consequently, according to the results obtained in [4, 5], the nature of the collapse of the cavity becomes unpredictable. Hence, in this paper an attempt is made to investigate the dynamics of the cavity generated by a contact electrical explosion experimentally in order to clarify the contribution of this stage of the process to the mechanism of the destructive action of the contact electrical explosion, and to find ways of improving technological processes which use this sort of electrical explosion.

For convenience we varied the length of the discharge gap l while keeping the remaining parameters fixed, viz., the breakdown voltage $U_0 = 50$ kV, the capacitance of the capacitor battery $C = 10^{-6}$ F, the inductance $L = 4.3 \times 10^{-6}$ H, the conductivity of the liquid $\sigma = 0.005$ $(\Omega \cdot \mathrm{m})^{-1}$, and the equivalent resistance of the circuit $R_e = 0.1$ Ω, defined from the curve of the current for the short-circuited discharge gap. The process was stabilized by initiating a discharge with a copper conductor of diameter 0.05 mm. The dynamic pattern of the development of the cavities was recorded with an SFR-2M high-speed motion-picture camera in a time loop using the method described in [6, 8], and was represented by a series of photographs (Fig. 1) as a function of the length of the discharge gap (on the right of each photograph we show the spatial scale b = 94 mm, and the exposure time ~0.2–0.4 msec).

Whereas when there is no contact surface the disintegration of the plasma cylinder in water with similar energy parameters is accompanied by its conversion into a pulsating cavity of quasispherical form [11], in the case of a contact explosion the evolution of this process is more complex: The formation of cavities is observed in the form of a spherical segment (see Figs 1a and b), dome-shaped (Figs. 1c–f) or a quasicylindrical shape (Fig. 1g).

The extremal amplitude–frequency parameters of the process are shown in the table (the number of the row of the table corresponds to the outer number of the series of photographs in Fig. 1, i.e., 1 corresponds to a, 2 corresponds to b, etc.).

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